CHAPTER 14

A CAPACITY EXPANSION PLANNING MODEL FOR MUNICIPAL WATER SUPPLY SYSTEMS

The model presented in this chapter is designed to provide a framework for balancing expected drought losses as a function of chosen adequacy level against the costs of improving that adequacy level. As mentioned above, it is in the tradition of inventory-adjustment/capacity-expansion models applied by others, although most such applications have taken deterministic forms. Because of the probabilistic nature of our capacity variable, there is no distinction in our model between under- and over-capacity, no line at which costs *begin* to be incurred. This feature, the fact that inadequacy is entirely a relative concept, results from the lack of any upper limit to drought severity. In principle, no matter how large a system's safe yield relative to its demand, a drought can occur that will be severe enough to cause shortage for that system.[1]

The choice variables controlled by the planners, subject to certain constraints, are the timing and sizes of increments to the system's safe yield. By choosing a time path of the level of system safe yield with given information about the rate of growth of demand, the planners determine a total present value of capital costs and a discounted sum of expected annual drought losses. The optimal plan for given growth rates and other parameters is the one which minimizes the total of these discounted costs and losses. We choose to work with a planning horizon of 60 years.

[1] In practical computations, of course, one has to draw the line on significant decimal places somewhere. If that line is drawn for probabilities at the fourth decimal place (numbers smaller than 1×10^{-4} are counted as zero), then any system with a D/Y ratio less than 0.3 is effectively drought-proof.

BASIC STRUCTURE OF THE MODEL

In Figure 21, we represent schematically the choice variables and resulting time paths of key variables for a particular plan choice. Note that we limit the decision variables to six: four increments, two of which must be constructed in years 0 and 60, and two at intermediate times. Each of these variables is continuous and, naturally, all are constrained to be non-negative. The times are further constrained to be less than 60.

In order both to make provision for the post-horizon future, and to standardize the set of possible paths, we introduce the constraint that at the end of year 60, the total safe yield constructed as part of the plan must equal the total growth in demand over the horizon. Thus, if we start with

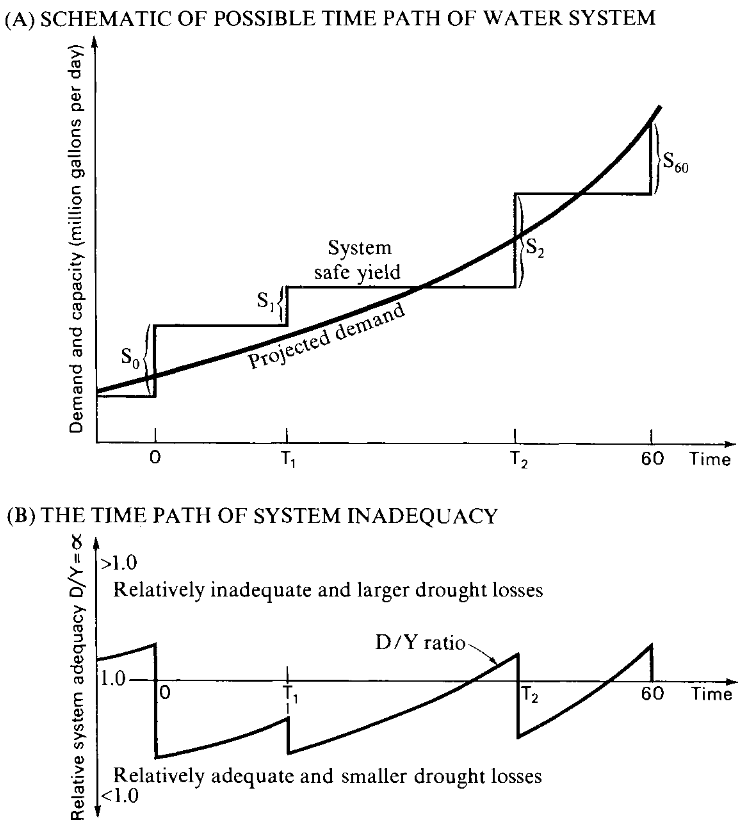

Figure 21. Schematic of choice variables and resulting time paths of key variables for a particular plan choice.

projected demand equal to safe yield in year zero, the same must be true at the end of year 60. This particular method of handling the bequest problem is recommended by its simplicity, but it also seems in some sense fair: we bequeath the same situation we have inherited in terms of our measure of relative adequacy.

FUNCTIONAL FORMS: A REVIEW AND CONSOLIDATION

We assume, first, that population and per capita demand grow exponentially according to the formulae:

$$\text{Population in year "}t\text{"} = N_t = N_o e^{\beta t} \tag{14-1}$$

$$\text{Projected Per Capita Demand}^2 \text{ in year "}t\text{"} = P_t = P_o e^{\gamma t} \tag{14-2}$$

We may thus express the total demand in year t as

$$D_t = N_t \cdot P_t = N_o P_o e^{(\beta+\gamma)t} = D_o e^{\alpha t} \tag{14-3}$$

where $N_o \cdot P_o = D_o$ and $\alpha = \beta + \gamma$.
The capital costs of safe-yield expansion are given by:

$$C(\Delta SY) = K(\Delta SY)^y$$

$$\text{or, if } \Delta SY \equiv s_t = s_0, s_1, s_2 \text{ or } s_{60} \tag{14-4}$$

$C(s_t) = K(s_t)^y$; (y generally < 1).

And so the present value of the cost of the increment s_t is:

$$PV[C \cdot (s_t)] = K(s_t)^y e^{-\rho t} \tag{14-5}$$

where ρ is the discount rate.

Now, the treatment of the expected losses from water shortage is, of necessity, more complex. From Chapter 12, we recall that expected annual per capita losses from drought for time t accrue at the rate

$$EAL_t = U[D_t / Y_t]^z \tag{14-6}$$

for given projected demand and chosen level of safe yield. In order to express this function in the terms we are presently employing, we define, $\bar{s}$ = level of safe yield inherited from the past. Then Equation 14-6 becomes:

$$EAL_t = U\left[\frac{D_o e^{(\beta+\gamma)t}}{\bar{s} + s_0 + \ldots}\right]^z \tag{14-7}$$

where $Y_t = \bar{s} + s_0 + \ldots$, and the exact composition of the denominator will depend on how many increments have been added prior to time t.

[2] We shall express both demand and safe yield in average daily terms. Our cost functions are standardized for arguments in millions of gallons per day (mgd). Hence, P_t is per capita daily demand in millions of gallons per day.

This implies that the *total* expected losses during a small period Δt, occurring at time t will be:

$$TEL_t = U\left[\frac{D_o e^{(\beta+\gamma)t}}{\bar{s} + s_0 + \ldots}\right]^z N_o e^{\beta t}\Delta t = UN_o\left[\frac{D_o}{\bar{s} + s_0 + \ldots}\right]^z e^{(\alpha z+\beta)t}\Delta t \tag{14-8}$$

where, we recall, $\alpha = \beta + \gamma$.

The present value of the total expected losses for the periods Δt at time t will then be:

$$PV(TEL_t) = UN_o\left[\frac{D_0}{s + s_0 + \ldots}\right]^z e^{(\alpha z+\beta-\rho)t}\Delta t \tag{14-9}$$

The next step is the computation of the total contribution to the present value of losses of a period between additions of safe yield. Let us, for example, consider the total present value of the losses suffered between time T_1 and T_2. Integrating, we have:

$$\text{Total } EL_{(T_2,T_1)} = \int_{T_1}^{T_2} UN_o\left(\frac{D_o}{\bar{s} + s_0 + s_1}\right)^z e^{(\alpha z+\beta-\rho)t}dt \tag{14-10}$$

which is particularly simple because t only appears in the exponent of e. Equation 14-10 gives us

$$\text{Total } EL_{(T_2,T_1)} = \left(\frac{UN_o}{\alpha z + \beta - \rho}\right)\left(\frac{D_o}{\bar{s} + s_0 + s_1}\right)^z \cdot [e^{(\alpha z+\beta-\rho)T_2} - e^{(\alpha z+\beta-\rho)T_1}]. \tag{14-11}$$

The objective function for the planning model may then be written in its entirety:

Total present value of costs and losses $= \theta(S_0, S_1, S_2, S_{60}, T_1, T_2)$

$$\begin{aligned} &= \sum_{t=0,T_1,T_2,60} K(s_t)e^{-\rho t} + \\ &+ \frac{N_o U(D_o)^z}{\alpha z + \beta - \rho}[(\bar{s} + s_0)^{-z}(e^{(\alpha z+\beta-\rho)T_1} - 1) \\ &+ (\bar{s} + s_0 + s_1)^{-z}(e^{(\alpha z+\beta-\rho)T_2} - e^{(\alpha z+\beta-\rho)T_1}) \\ &+ (\bar{s} + s_0 + s_1 + s_2)^{-z}(e^{(\alpha z+\beta-\rho)60} - e^{(\alpha z+\beta-\rho)T_2})]. \end{aligned} \tag{14-12}$$

The constraint set includes:[3]

$$\sum_{t=0,T_1,T_2,60} S_t = D_o(e^{60\alpha} - 1) \tag{14-13}$$

[3] If we wish to allow for an inequality we may write:

$$s_0 + s_1 + s_2 + s_{60} - s_s = PD_o(e^{60\alpha} - 1)$$

where s_s is a slack variable and $s_s \geq 0$. This formulation might be useful where demand is growing slowly or not at all and some capacity above the minimum required to cover that growth might reduce drought losses by enough to be worth while.

$$S_t \geq 0 \text{ all } t$$

and,[4]

$$0 \leq T_1 \leq 60,$$
$$0 \leq T_2 \leq 60,$$

The following constants retained the values indicated for all runs:[5]

$$N_o = 10^5$$
$$D_o = 10(\text{mgd})$$
$$P_o = 10^{-4}(\text{mgd})$$

and,

$$K = 1.28 \times 10^5;$$
$$U = 0.1.$$

GENERAL COMMENTS

The capital-cost portion of the objective function is concave (since $y < 1$). Thus, without consideration of drought losses, the minimum-cost plan would be the construction of the entire required increment in the final year.[6] When, however, we include drought losses the situation becomes at once more complicated and more promising for meaningful minimization solutions.

METHODS OF SOLUTION

Two methods were used in finding the solutions to this programming problem. First, as a method of developing some familiarity with the behavior of the solutions, a rather simple search technique was employed. Then, a nonlinear programming algorithm based on the method of

[4] The objective function as formulated above depends on T_1 being less than T_2. It might seem that it would be necessary to add a constraint to this effect; for example, writing $T_1 + T_s = T_2$, with $T_s \geq 0$, a slack variable. In practice, however, this problem did not arise in any of our many solution runs, and we have not included such a constraint. One conjectures that since one increment must necessarily be added later than the other (unless $T_1 = T_2$) the problem is only one of labeling.

[5] As noted later, a few runs were made with $N_o = 50{,}000$ in connection with comparison of our optimality "rules of thumb" and the actual expansion histories of three Massachusetts systems.

[6] Since there is effectively one constraint, the extreme point vector will have only one non-zero element. This implies the economically sensible result that either s_0 or s_{60} must be non-zero, since the other four variables are, in a sense, paired. At a positive discount rate, everything will be built in year 60; with a zero discount rate, the planner would be indifferent between the first and last years.

Zoutendijk was applied.[7] These methods are both discussed in somewhat more detail in Appendix H, while the actual results are discussed below.

RESULTS OF SOLUTION OF THE MODEL FOR VARIOUS PARAMETER VALUES

We are interested here in changes in total costs and in the elements of the solution vector in response to changes in the scaling factor, the discount rate (ρ), and the key drought loss-function parameter (z). The combinations of parameters for which the model was solved are listed in Table 39.

TABLE 39. COMBINATIONS OF INITIAL VECTORS AND PARAMETER VALUES USED IN COMPUTER RUNS

Initial vectors	Loss function parameter z	Safe yield scale factor y	Discount rate ρ	Population rate of growth β	Per capita daily consumption rate of growth γ
1 2 3	3.2 4.3 5.4 12.0	0.88 0.78 0.68	0.07 0.05 0.03	0.015	0.020
					(108 runs total)
2 3	4.3 12.0	0.78	0.05	0.000 0.015 0.030	0.000 0.020 0.040
					(36 runs total)

The "best" solution vectors and costs are shown in Table 40 for the 36 combinations of y, ρ, and z. We report here the total costs, capital costs, and drought losses to the nearest thousand dollars. Times (T_1 and T_2) are reported to the nearest tenth of a year and capacity increments to the nearest hundredth mgd.[8] ($\beta = 0.015$; $\gamma = 0.020$.)

[7] See G. Zoutendijk, *Methods of Feasible Directions* (Amsterdam: Elsevier, 1960).
[8] For a comparison of the results found using the search technique with the programming solutions, see Appendix H.

TABLE 40. SENSITIVITY OF RESULTS TO PARAMETER CHANGES

(costs given in $1,000)

Z	Y	ρ	Total cost	Capital cost	Drought loss	T_1	T_2	S_0	S_1	S_2	S_{60}
12.0	0.88	0.07	1,339	1,182	157	15.2	34.5	4.52	12.66	34.78	19.71
12.0	0.88	0.05	1,898	1,723	175	17.4	37.4	5.99	15.04	34.95	15.67
12.0	0.88	0.03	2,922	2,743	178	20.1	40.0	8.12	18.06	35.68	9.80
12.0	0.78	0.07	1,088	971	117	15.2	34.6	4.81	13.18	36.40	17.27
12.0	0.78	0.05	1,497	1,372	125	17.5	37.7	6.37	15.87	36.79	12.63
12.0	0.78	0.03	2,214	2,130	84	20.8	42.8	9.59	21.28	40.80	0
12.0	0.68	0.07	883	799	84	15.7	35.3	5.37	14.13	37.76	14.41
12.0	0.68	0.05	1,179	1,092	87	18.7	39.5	7.32	18.04	37.88	8.42
12.0	0.68	0.03	1,712	1,623	89	26.0	53.0	13.54	32.98	25.63	0
5.4	0.88	0.07	1,145	838	307	14.8	34.8	2.14	10.11	25.90	33.51
5.4	0.88	0.05	1,727	1,382	345	17.1	37.5	3.69	12.31	27.36	28.30
5.4	0.88	0.03	2,836	2,461	374	19.4	40.1	5.85	15.20	30.02	20.60
5.4	0.78	0.07	980	734	246	14.5	34.9	2.40	11.11	28.27	29.88
5.4	0.78	0.05	1,419	1,156	263	17.3	38.2	4.22	13.75	30.26	23.44
5.4	0.78	0.03	2,214	2,031	183	22.5	46.4	9.30	22.42	39.94	0
5.4	0.68	0.07	835	651	184	15.6	36.4	3.28	12.85	30.76	24.78
5.4	0.68	0.05	1,159	970	189	18.8	40.3	5.48	16.41	33.44	16.33
5.4	0.68	0.03	1,697	1,508	189	23.3	46.4	8.80	25.06	37.80	0
4.3	0.88	0.07	1,059	602	457	13.2	34.3	0	9.52	23.38	38.76
4.3	0.88	0.05	1,655	1,239	417	16.5	37.2	2.56	11.26	24.65	33.19
4.3	0.88	0.03	2,794	2,339	455	19.0	39.9	4.90	14.17	27.68	24.91
4.3	0.78	0.07	906	504	402	12.8	34.1	0	10.92	22.77	37.97
4.3	0.78	0.05	1,382	1,052	329	16.1	37.7	2.82	12.77	27.94	28.13
4.3	0.78	0.03	2,203	1,960	243	23.8	48.5	8.88	23.49	39.29	0
4.3	0.68	0.07	798	392	406	14.2	37.5	0	13.59	29.99	28.08
4.3	0.68	0.05	1,141	923	218	20.9	45.8	5.36	20.15	46.15	0
4.3	0.68	0.03	1,712	1,528	185	24.5	47.0	9.93	26.05	35.68	0
3.2	0.88	0.07	948	426	522	19.9	44.1	0	12.20	23.63	35.83
3.2	0.88	0.05	1,523	920	603	15.5	38.6	0	10.76	22.05	38.85
3.2	0.88	0.03	2,711	2,005	706	11.5	34.0	0	10.26	26.14	35.26
3.2	0.78	0.07	836	377	459	19.4	43.5	0	14.22	32.28	25.16
3.2	0.78	0.05	1,294	813	480	15.8	41.6	0	15.39	34.09	22.18
3.2	0.78	0.03	2,147	1,732	415	7.8	36.4	0	14.86	32.08	24.72
3.2	0.68	0.07	708	312	396	16.3	37.7	0	11.39	21.98	38.29
3.2	0.68	0.05	1,088	546	492	16.5	42.2	0	17.51	25.64	28.51
3.2	0.68	0.03	1,708	1,392	317	22.4	47.7	6.10	24.00	41.57	0

By far the most striking of our findings is the insensitivity of total losses to changes in z. We note, for example, that when $y = 0.68$ and $\rho = 0.03$, the difference in total costs between the $z = 12$ and the $z = 3.2$ situations is only $4,000 or about 0.18 percent. It is not true, of course, that we can dismiss the problem of determining the proper z, for the similarity in total costs noted above was only achieved by following quite different optimal paths. In particular, if z is large, we wish to build considerably more capacity in the initial year (13.54 mgd vs. 6.10 mgd in our example).[9] The remaining elements of the two plans are not very different, though $\hat{S}_1$ is also larger when $z = 12$.

Indeed, another interesting feature of the solution data is the relative insensitivity of these remaining elements of the optimal plan to changes in z. For example, for $y = 0.78$, $\rho = 0.05$, as z changes from 12 to 3.2, the timing portion of the solution vector changes only from $T_1 = 17.5$, $T_2 = 37.7$ to $T_1 = 15.8$, $T_2 = 41.6$. We may contrast this with the change in timing for $z = 5.4$, $y = 0.78$ as ρ goes from 0.07 ($T_1 = 14.5$, $T_2 = 34.9$) to 0.03 ($T_1 = 22.6$, $T_2 = 46.4$). Similar, if less pronounced, differential sensitivity may be observed in the capacity elements of the solution vectors.

The observed differences in total plan cost between $z = 12$ and $z = 3.2$ situations are considerably greater under other combinations of y and ρ. In general these differences seem to be greater, the greater y and ρ. When $y = 0.88$ and $\rho = 0.07$, the cost difference is about $400,000, or 30 percent. But even this difference is small compared to those observed as ρ varies from 0.07 to 0.03 for given z and y. For example, when $z = 5.4$, and $y = 0.88$, the difference between total costs at 3 percent and at 7 percent discounting is about $1,700,000, or 60 percent of the larger figure (150 percent of the smaller). *It seems clear that both total costs and the values of* $\hat{T}_1$, $\hat{T}_2$, *etc. are most sensitive to the discount rate.*

The heavy impact of this parameter is not surprising. When z is large, the model seeks a path which includes a bit more capacity early in the period to keep the D/Y ratio down in the area around 1.00 for which drought losses per capita are small even for large z.[10] But when ρ is smaller, whatever the optimal path involves it is bound to cost more, since ρ acts symmetrically on capital costs and drought losses. The only trade-offs

[9] The cost of misspecification of z may be found for two situations: if we build the plan optimal for $z = 3.2$ and z is "really" 12, then we lose over $740,000 in increased drought losses. If we build for $z = 12$ when it is really 3.2, we lose about $75,000. This represents the net of $230,000 in increased capital costs and $155,000 in reduced drought losses. See below for further evidence on the importance of z.

[10] This strategy results in lower total drought losses, for given y and ρ when $z = 12$ than when $z = 3.2$, 4.3, or 5.4. The somewhat higher total costs for larger z reflect the higher present value of capital costs implied by building early.

possible here involve time, for postponement either of capital costs or of losses is less well rewarded under a smaller discount rate.

We note that the model's sensitivity to changes in ρ extends more deeply than a mere increase in the present value of total expected costs and losses. When ρ changes, so do the two elements of the plan vector particularly important to planners; i.e., the size of the increment to be built in year zero and the time to the next increment. For the parameter combination $z = 5.4$, $y = 0.88$, when ρ falls from 0.07 to 0.03, $\hat{S}_o$ increases from 2.14 to 5.85, or by well over 100 percent.

It is interesting that *while total costs show some sensitivity to changes in y, the composition of the optimal path shows virtually none at all.* Thus, for example, when $z = 4.3$, and $\rho = 0.07$, as y changes from 0.68 to 0.88, total costs increase from \$798,000 to \$1,059,000 or by about 25 percent of the lower figure. But, over this same interval, the optimal vector changes very little in terms of the generally sensitive variables $\hat{S}_o$ and $\hat{T}_1$.[11]

By way of tentative summary we may suggest that an accurate estimate of the appropriate interest rate is at least as important for effective planning as exact knowledge of either the loss function or the scaling factor (given that the latter are in the range we use). This is true even though we realize that in determining the optimal plan, the planners need make no irrevocable decisions except what size to make S_o.

[11] As discussed below, we had hoped that in each run the solution vectors found using the three different initial vectors would be the same. Had this invariably happened, we would have been reasonably confident that we were finding the global minimum. Unfortunately, however, we did not achieve such ideal results. Indeed, of the 36 parameter sets solved with separate initial vectors, only 12 (about 33 percent) gave results which could be classified as triple agreement on a single optimum. Three different kinds of problems could be distinguished in the 24 cases of nonagreement, all resulting from the nonconvexity of the surface. First, for certain parameter combinations, the partial derivatives show perverse sign behavior in the neighborhood of one or more of the starting vectors. In these instances the program reports as the solution a minor modification of the particular starting vector. (These "solutions" are recognizable even without knowledge of the global optimum and there is very little danger of their acceptance in an actual series of computations.) Second, when one of the increment sizes naturally tends to be small because of the particular parameter combinations, corner solutions again turn up. This is particularly troublesome in the fairly large number of cases for which s_o tends to zero. Here no solution is obviously incorrect, but all are highly suspect and only the performance of a large number of runs gives any assurance that the solution accepted is close to the best attainable. Third, what appear to be classical local optima within the constraint region are occasionally turned up.

Generally, the method worked best when z was 5.4. For large z, the program tended to find starting-vector corner solutions when ρ and/or y were small; when ρ and y were large the program moved more securely to a single optimum. When z was small (3.2), the problem created by the tendency of s_o to zero cropped up frequently. We may place the most faith in the results found for $z = 5.4$ and 4.3, the least on the results for $z = 3.2$.

We move on now to examine the sensitivity of the solutions to changes in the growth rate of demand and to investigate the implications for the planning process of uncertainty about these rates of growth. We shall note that good estimates of these rates are more critical to effective planning than is knowledge of any of the other key parameters.

EFFECT OF UNCERTAINTY IN PROJECTIONS OF THE GROWTH OF DEMAND

Our first step is to look at the solutions to the planning model under 9 combinations of population and demand rates of growth. We then calculate the costs of adopting programs optimal under one set of growth rates, when the actual rates of growth are different. The calculations are performed under two sets of assumptions, one of which represents an extreme view of the possible losses to be incurred, and the other of which is an attempt to capture a more realistic set of losses. The difference between the two cases is the time at which we assume the growth-rate discrepancy is discovered and a return to the optimal path effected.

OPTIMAL PATHS AND RESULTING COSTS UNDER VARIOUS SETS OF GROWTH RATES

In Table 41 we summarize the values of costs and of the choice variables under 9 different combinations of assumed growth rates of population and per capita demand. We report the results for two different sets of basic parameters: (i) $z = 12$, $y = 0.78$, $\rho = 0.05$; and (ii) $z = 4.3$, $y = 0.78$, $\rho = 0.05$.

These results do not hold any particular surprises, although they do illustrate that the total costs are relatively sensitive to changes in growth rates. Thus, let us compare the results for an overall growth rate of demand of 0.015 with those for a rate of 0.070. This represents an increase in the growth rate of about 4.7 times. When $z = 4.3$, this produces a difference in costs of 7.3 times, and when $z = 12$, of 9.8 times. Drought losses do not vary nearly so much over this range—only by factors of about 2 in each case. The large differences in total costs thus reflect proportionally even greater changes in capital costs (on the order of 10 or 11 times). These increases are, in turn, based on the fact that total demand growth to be covered by construction increases far more than in proportion to the increase in the rate of growth. (As the growth rate goes from 0.015 to 0.070, the total growth of demand over 60 years increases by a factor of 45.) The capital cost increases are, however, dampened by two factors. First, of course, economies of scale hold down the costs of build-

TABLE 41. SENSITIVITY OF RESULTS TO GROWTH RATE CHANGES

ζ	β	γ	Total cost	Capital cost	Drought loss	T_1	T_2	S_0	S_1	S_2	S_{60}
12.0	0.000	0.000	180	80	100	—	—	5.47	—	—	—
12.0	0.015	0.000	633	540	93	27.1	46.3	3.88	5.54	5.18	—
12.0	0.030	0.000	1,275	1,161	114	17.6	38.0	5.55	13.18	28.50	3.27
12.0	0.000	0.020	764	676	88	16.7	42.2	3.02	7.21	12.97	—
12.0	0.015	0.020	1,497	1,372	125	17.5	37.8	6.37	15.87	37.03	12.40
12.0	0.030	0.020	2,736	2,549	188	20.1	41.9	13.31	46.40	93.45	37.77
12.0	0.000	0.040	1,753	1,610	143	18.4	39.6	7.56	21.64	45.90	25.13
12.0	0.015	0.040	3,167	2,953	214	20.4	41.6	15.30	53.50	120.71	71.62
12.0	0.030	0.040	6,181	5,965	216	27.4	53.6	42.13	299.59	283.48	31.66
4.3	0.000	0.000	190	—	190	—	—	—	—	—	—
4.3	0.015	0.000	659	343	315	8.6	51.4	—	4.84	7.30	2.46
4.3	0.030	0.000	1,274	957	317	16.3	38.4	2.67	12.33	26.53	8.95
4.3	0.000	0.020	712	378	334	10.8	49.2	—	5.39	11.16	6.21
4.3	0.015	0.020	1,382	1,052	329	16.1	37.7	2.82	12.77	27.94	28.13
4.3	0.030	0.020	2,470	2,055	415	18.3	39.5	6.97	29.51	88.23	66.15
4.3	0.000	0.040	1,500	1,161	339	16.2	37.5	3.09	13.50	29.79	53.86
4.3	0.015	0.040	2,685	2,253	432	18.3	39.3	7.28	31.06	93.98	128.81
4.3	0.030	0.040	4,817	4,235	582	22.7	45.1	17.41	106.00	231.04	302.42

ing more capacity. And, second, under the higher growth rate relatively less of the total required construction is undertaken in the first year of the period. (Under the 0.015 growth rate, 26.6 percent of the total construction is carried out in year zero. When the overall growth rate is 0.070, only 6.4 percent is done in the first year.)

Losses From Incorrect Demand Projections: The Case of the Obtuse Planners

If we assume that planners are faced with a range of possible growth rates for population and per capita demand, we may ask what kinds of losses (excess costs) are involved in acting on the basis of one such combination when another combination, in fact, describes the state of nature over the planning period. In order to obtain an estimate of the largest possible losses for a given strategy, given parameter values, and given range of growth rates (and given that the bequest constraint is met), we first assume that the planners involved are extremely obtuse: so much so that they proceed with the chosen plan right through the period, ignoring any evidence of a discrepancy between the assumed and actual rates of growth. Only in year 60 do they realize their mistake. At that time they build, if it is necessary, an increment large enough to meet the bequest constraint.

For simplicity, we consider the set of possible growth-rate combinations to include only the following 9 elements:

Population Growth, (β)	0.000	0.015	0.030	0.000	0.015	0.030	0.000	0.015	0.030
Per Capita Demand Growth, (γ)	0.000	0.000	0.000	0.020	0.020	0.020	0.040	0.040	0.040
Total Demand Growth	0.000	0.015	0.030	0.020	0.035	0.050	0.040	0.055	0.070

[We assume $y = 0.78$ and $\rho = 0.05$, in what follows. Results are reported for $z = 4.3$ and $Z = 12$. We futher assume that a single rate of growth holds over the entire period, whether this rate is the one assumed by the planners or not.]

We assume that the planners accept the pair $\beta = 0.015$ $\gamma = 0.020$, as the best estimate of the future growth rates, and that they act to follow the plan optimal for this pair. In Table 42 we show the losses resulting from such a policy when each of the other growth rates in fact turns up. These losses represent the difference between the actual costs implied by the assumed strategy and those which would have been incurred under

TABLE 42. RESULTS OF NON-OPTIMAL POLICIES, WITHOUT REVIEW

(all costs expressed in $1,000)

Actual growth rates β	γ	Total cost: Actual	Total cost: Difference from optimal	Capital cost: Actual	Capital cost: Difference from optimal	Drought losses: Actual	Drought losses: Difference from optimal
$Z = 4.3$							
0.000	0.000	1,010	820	970	970	40	(150)
0.015	0.000	1,050	392	970	627	80	(235)
0.030	0.000	1,280	6	1,000	43	280	(37)
0.000	0.020	1,050	348	970	592	90	(244)
0.015	0.020	1,380	—	1,050	—	330	—
0.030	0.020	7,540	5,070	1,280	(775)	6,260	5,845
0.000	0.040	1,510	10	1,120	(41)	390	51
0.015	0.040	9,960	7,275	1,390	(863)	8,570	8,138
0.030	0.040	577,000	572,200	1,920	(2,300)	575,000	574,500
$Z = 12.0$							
0.000	0.000	1,330	1,150	1,330	1,250	0	(100)
0.015	0.000	1,332	699	1,330	790	2	(91)
0.030	0.000	1,370	95	1,330	169	40	(74)
0.000	0.020	1,334	570	1,330	654	4	(84)
0.015	0.020	1,500	—	1,370	—	130	—
0.030	0.020	1.9×10^6	1.9×10^6	1,610	(900)	1.9×10^6	1.9×10^6
0.000	0.040	2,070	317	1,440	(170)	630	487
0.015	0.040	2.6×10^7	2.6×10^7	1,730	(1,200)	2.6×10^7	2.6×10^7
0.030	0.040	2.4×10^{12}	2.4×10^{12}	2,260	(3,700)	2.4×10^{12}	2.4×10^{12}

Note: Figures in parentheses indicate savings.

certainty by using the plan appropriate to the actual growth rate. As such, they may be thought of as costs of uncertainty.

We note from Table 42 that the results of such doggedly incorrect decision-making could be literally disastrous. We note, however, that the magnitude of the problem varies enormously with the size of z, the drought loss function. It is in this area of uncertain demand projections that we find the first signs that the planning process is seriously sensitive to z. Clearly it is important to know what z is in order to choose among alternative strategies for dealing with this problem.

Losses From Incorrect Demand Projections: A More Realistic Case

Let us consider the same combination of parameter values and the same assumptions about the range of possible growth rates. In this example, however, we assume that the planners act so as to retain some flexibility in the face of uncertainty about these rates. Specifically, before building the increment $\hat{s}_1$, optimal for $\hat{T}_1$ under the assumed growth rates $\beta = 0.015$ and $\gamma = 0.020$, the planners check these assumed rates against evidence from the world. Let us assume that the evidence indicates that the actual rates of growth are $\beta = b$ and $\gamma = g$. (We assume that these rates have, in fact, held since the year 0 and will hold to the horizon.) With these rates of growth and given the information about z, y and ρ, there is associated an optimal plan vector, T_{1bg}, T_{2bg}, s_{0bg}, s_{1bg}, s_{2bg}, s_{60bg}. The planners, acting on their discovery, build increment s_{1bg} in time $\hat{T}_1$, instead of continuing with their original plan. Subsequently they build s_{2bg} in time T_{2bg}, and an amount in year 60 determined by the bequest constraint.

TABLE 43. RESULTS OF NON-OPTIMAL POLICIES, WITH REVIEW

(all costs expressed in $1,000)

Actual growth rates		Total cost		Capital cost		Drought losses	
β	γ	Actual	Difference from optimal	Actual	Difference from optimal	Actual	Difference from optimal
Z = 4.3							
0.000	0.000	353	163	287	287	65	(125)
0.015	0.000	695	36	529	186	166	(149)
0.030	0.000	1,270	0	970	15	300	(15)
0.000	0.020	759	47	590	212	169	(165)
0.015	0.020	1,380	0	1,050	0	329	0
0.030	0.020	2,560	90	1,850	(205)	710	295
0.000	0.040	1,500	0	1,140	(30)	360	30
0.015	0.040	2,800	115	2,030	(220)	760	335
0.030	0.040	5,520	705	3,970	(270)	1,560	975
Z = 12.0							
0.000	0.000	543	363	542	462	1	(99)
0.015	0.000	800	167	791	251	9	(84)
0.030	0.000	1,290	15	1,220	59	70	(44)
0.000	0.020	916	152	906	230	10	(78)
0.015	0.020	1,497	0	1,372	0	125	0
0.030	0.020	3,740	1,000	2,270	(275)	1,460	1,275
0.000	0.040	1,770	17	1,560	(50)	210	67
0.015	0.040	5,400	2,333	2,600	(353)	2,800	2,233
0.030	0.040	62,400	56,209	25	5,990	56,400	56,184

Note: Figures in parentheses indicate savings.

The results for this example are contained in Table 43. We note that, with one exception, the total costs associated with early adjustment to incorrect demand projections are of the same order of magnitude as those resulting from proper projections. Only in the situation for which $z = 12$, and the overall growth rate of demand is, in fact, 0.070 (i.e., when $\beta = 0.030$, $\gamma = 0.040$) does a really larger loss show up. This observation puts into better perspective the question of over-building as a hedge against uncertainty. *The indication is that only in relatively unusual circumstances would even a serious underestimate of demand growth imply very large losses, if the planners take the reasonable precaution of checking their assumptions.*

With the above evidence in mind on the existence of persuasive reasons for a bias toward "overbuilding" of water supply systems, it will be particularly interesting to turn to a consideration of the practical planning process. In the next chapter, we discuss rules of thumb, based on our computational results, for use in the making of municipal water supply decisions.